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Supporting Materials

Line active hybrid lipids determine domain size in phase separation of saturated and unsaturated lipids

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Supporting Materials for "Line Active Hybrid lipids Determine Domain Size in Phase Separation of Saturated and Unsaturated Lipids"

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INTERFACE CONCENTRATION

As in ref. (5) we assume a simple model where for strong interactions (relative to k_BT), the bulk is composed mostly of saturated lipids, unsaturated lipids and cholesterol. The saturated and unsaturated lipids occupy the nodes of a square lattice and the cholesterol is assumed to only affect the ordering of the lipid chains, which in turn affects the interactions. To this system, we consider the addition of a small concentration of a hybrid lipid which, as described in the text, interacts like a saturated lipid on one end and an unsaturated lipid on the other. This hybrid lipid takes up only a small fraction of the overall volume and our model therefore considers it at interstitial positions in the lattice-gas model for the saturated-unsaturated system. The interactions are written as J_{ss}, J_{uu} and J_{su} which represents interactions between saturated-saturated, unsaturated-unsaturated and saturated-unsaturated chains, respectively. The total mean-field free energy (ref. (5)) of such a system as a function of local saturated concentration ψ is,

$$g = f_0 + f_h + \frac{J_T(a\nabla\psi)^2}{4} - \mu\psi - \mu_h\psi_h, \tag{S.1}$$

where the chemical potentials of the hybrid and the saturated lipids are μ_h and μ respectively. f_0 is the free energy (interactions and entropy) of the basic saturated/unsaturated system and f_h is the free energy of the hybrid lipid that resides on a fraction, ψ_h, of the interstitial sites. These two contributions can be written,

$$f_0 = T\left[\psi\log(\psi) + (1-\psi)\log(1-\psi)\right] + \frac{1}{2}\left[J_T\psi(1-\psi) - \Delta J\psi - J_{uu}\right], \tag{S.2}$$

and

$$f_h = T\left[\psi_h\log(\psi_h) + (1-\psi_h)\log(1-\psi_h) - \psi_h\log\left(\cosh\left[\frac{J_T(a\nabla\psi)}{2T}\right]\right)\right] - \frac{\psi_h}{2}\left[J_T\psi(1-\psi) - \frac{\Delta J}{2}(\psi-\frac{1}{2})\right]. \tag{S.3}$$

The interaction parameter $J_T = J_{ss} + J_{uu} - 2J_{su}$ should be interpreted as the energetic benefit of reducing the amount of interface with an consequent increase in the number of molecules (and interactions) in the bulk. The second

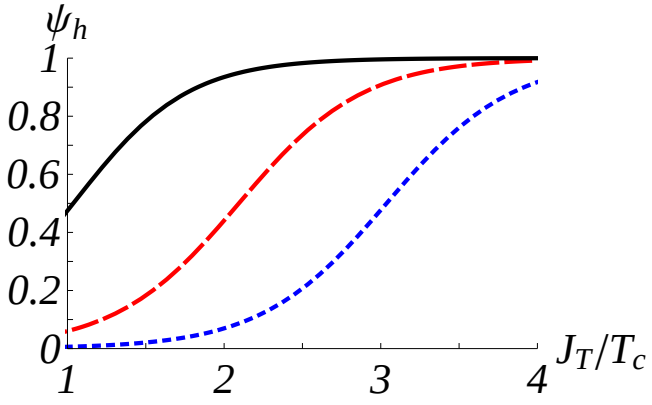

Fig. S1: Interfacial concentration of hybrid lipid vs interaction strength of lipid components for overall hybrid concentration ranging over three orders of magnitude ($\bar{\psi}_h = 0.001$ blue dotted, $\bar{\psi}_h = 0.01$ red dashed, $\bar{\psi}_h = 0.1$ black solid). Even for very small concentrations of hybrid lipid, the interface saturates for interaction strengths two or three times greater than the critical temperature. To first order in $\bar{\psi}_h$, the critical temperature, $T_c = J_T(1-\psi_h)/4$ in our lattice-gas model.

interaction parameter $\Delta J = J_{ss} - J_{uu}$ is the energetic difference between the saturated-saturated and unsaturated-unsaturated interactions which, for our purposes, is taken to be negligible; see ref. (5). In Eq. S.3, the first two terms are the lattice-gas entropy of the hybrid and the last term in square brackets includes the interactions of the hybrid with the saturated and unsaturated lipids (see Ref. (5)). The middle term that contains the composition gradient arises from the orientational entropy of the hybrid and its coupling to the gradient of composition of the saturated-unsaturated lipids (see Ref. (5) for the derivation). This term is responsible for the accumulation of hybrid at the interface and the consequent reduction of the line tension by the hybrid since this term is always negative and increases as the composition gradient increases.

In the accompanying letter, we assume that when the saturated/unsaturated phase separation is complete, the interface is saturated with hybrid lipid. This claim can be verified by examining the local hybrid concentration at the interface, $\psi_h^{(i)}$, found by minimizing the above expression for the free energy with respect to ψ_h and using the relationship between the hybrid chemical potential, μ_h, and the average hybrid fraction, $\bar{\psi}_h$ (see Ref. (5)). The result is given as a function of total hybrid concentration and interaction parameter J_T,

$$\psi_h^{(i)} = \frac{1}{1 + \frac{(1-\bar{\psi}_h)}{\bar{\psi}_h} \cosh\left(\frac{J_T}{2T}\right) \exp\left[-\frac{J_T}{8T}\right]}. \tag{S.4}$$

In the large interaction limit where the coexisting bulk phases are approximately characterized by $\psi \approx 1$ and 0 (nearly pure saturated and pure unsaturated phase), the composition approaches these limits with corrections of $\exp\left[-2J_T/T_c\right]$. It is easy to see that the assumption of distinct domains with a well defined, sharp interface (that occur when $J_T/T_C >> 1$) implies the interface has already begun to saturate with hybrid for any reasonable overall hybrid concentrations, $\bar{\psi}_h$ as shown in Fig. S1 ($\bar{\psi}_h = 0.001$ blue dotted, $\bar{\psi}_h = 0.01$ red dashed, $\bar{\psi}_h = 0.1$ black solid).

BENDING FREE ENERGY

The parameter J_T contains contributions from chain packing entropy, Van der Waals forces and headgroup interactions. However, in Eqn. 1 of the main text the free energy contribution from bending is isolated from the larger contributions that control the phase behavior and the value of the interfacial line tension for a flat interface. This is because the chain packing entropy (for fixed headgroup packing) difference between the saturated and unsaturated chains of the hybrid make the line energy sensitive to curvature. That is, the relative chain packing entropy is responsible for the bending energy (and the spontaneous curvature of the interface), while the other interactions are the major contributions to the reduction of the line tension, even for flat interfaces. The overall contribution to the mean-field interaction parameter from the bending portion is $J_T^b = 2k(L_s - L_u)^2/a_0$, where a_0^{-1} is simply the number of hybrid molecules per unit length at the interface. If k is a typical energy scale of $\sim 1\ k_BT$/nm then $J_T^b \approx (1/10)k_BT$ and we do not expect this bending contribution to strongly influence the bulk phase behavior or the reduction of the line tension by hybrid. However, it is this term that causes the free energy to be sensitive to the curvature of the interface and hence determines the optimal size of the domains that are stabilized by the hybrid lipids.